AN ACCURACY ANALYSIS OF AN AD HOC LOWER CONFIDENCE LIMIT PROCEDURE FOR SYSTEM AVAILABILITY

Bjørnar Johan Kibsgaard



United States Naval Postgraduate School



THESIS

AN ACCURACY ANALYSIS OF AN AD HOC
LOWER CONFIDENCE LIMIT PROCEDURE
FOR SYSTEM AVAILABILITY

by

Bjørnar Johan Kibsgaard

Thesis Advisor:

W. M. Woods

September 1971

Approved for public release; distribution wilimited.



An Accuracy Analysis of an Ad Hoc Lower Confidence Limit Procedure for System Availability

by

Bjørnar Johan Kibsgaard Lieutenant Commander, Royal Norwegian Navy Royal Norwegian Naval Academy, 1962

Submitted in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL

September 1971



LIBRARY NAVAL POSTGRADUATE SCHOOL MONTEREY, CALIF. 93940

ABSTRACT

The accuracy of proposed lower confidence limits for system availability is analyzed. Random values of the lower 100(1-)% confidence limit ${}^{A}_{SL}()$ for system availability are computed for a system whose failure density is exponential () and whose repair density is exponential (). The system is modeled as an alternating renewal process. The 100(1-) th percentile point of the generated distribution of ${}^{A}_{SL}()$ is compared with system availability as a measure of accuracy for ${}^{A}_{SL}()$.



TABLE OF CONTENTS

I.	INI	ROD	OUCTION	4				
II.	STA	ATEN	MENT OF THE PROBLEM AND ANALYSIS	5				
	Α.	DEF	FINITION OF AVAILABILITY	5				
	в.	PRO	DBLEM	8				
	c.	ALT	CERNATIVE METHOD	9				
	D.	PRO	OCEDURE	11				
	E.	ANA	ALYSIS	13				
III.	AC	CURA	ACY RESULTS	14				
APPEI	NDIX	A:	RESULTS OF THE SIMULATION	16				
APPEI	NDIX	В:	FORTRAN IV COMPUTER PROGRAM	17				
APPEI	NDIX	C:	DERIVATION OF MEAN AND VARIANCE OF \hat{A}_S	19				
BIBLIOGRAPHY								
INITIAL DISTRIBUTION LIST								
FORM DD 1473								



I. INTRODUCTION

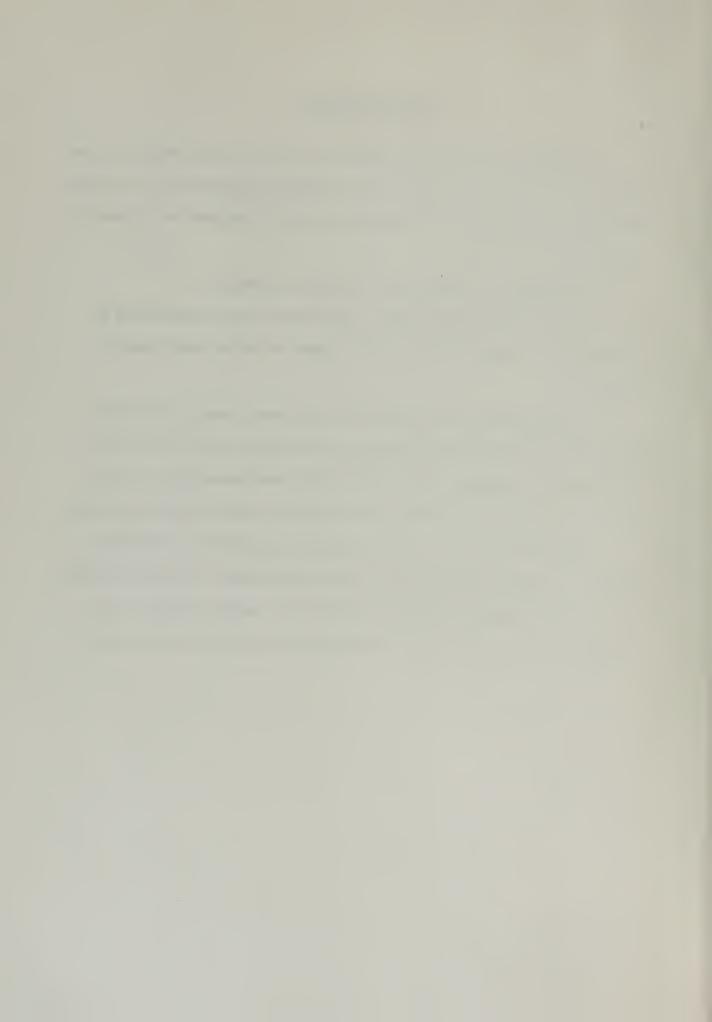
The system considered in this thesis is of the type which operate for a time, fail, are repaired and returned to operation. The system then may be said to have two states and can be modeled as a renewal process.

The objectives of this paper were the following:

- 1. To compute exact values of the steady state availability for a system with given distributions of times to failure and times to repair.
- 2. To compare the availability of a system based on the test procedures used with the 100(1-) th percentile point of the distribution of a proposed 100(1-) lower confidence limit. This will allow an accuracy analysis of a proposed lower confidence limit.

The method used was to fit a normal distribution to simulated data and compute the 100(1-x)) th percentile point of the distribution.

An alternative method was investigated, which fitted a 2 parameter gamma distribution. The procedure proved to be inaccurate.



II. STATEMENT OF THE PROBLEM AND ANALYSIS

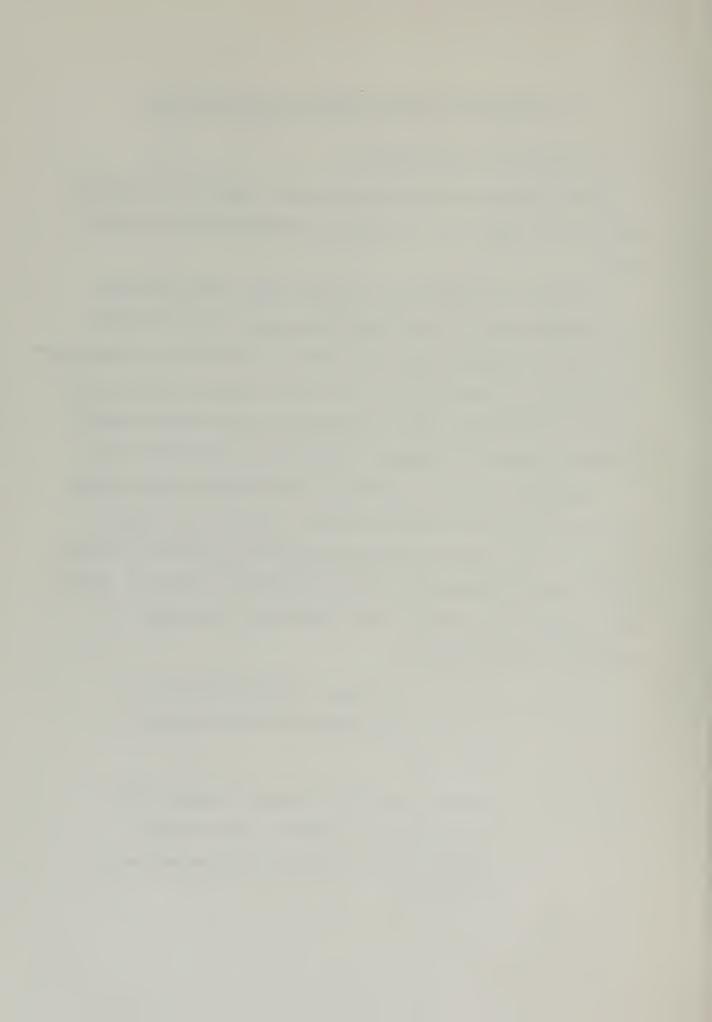
A. DEFINITION OF AVAILABILITY

In this paper the term availability [1] is defined as the probability that a system will be "up" or operable when called upon at a random time.

Normally, availability is not specified alone. Both reliability and maintainability are usually specified together with availability. For durable, continuous operated hardware, reliability can be specified by mean-time-to-failure (MTTF). Given the exponential form of time to failure distribution, which is applicable to most types of complex equipment, MTTF is a constant, the reciprocal of the failure rate.

Maintainability can be specified by mean-time-to-repair (MTTR) and mean-preventive-maintenance-time. Corrective maintenance times are often found to be well described by the exponential distribution. Given that the time-to-repair is distributed exponential, MTTR is a constant, the reciprocal of the repair rate. Accordingly we make the following definitions.

- (1) A(t) Availability at time t, or the probability that an item will be operable at a stated instant in time. $t \ge 0$
- (2) \overline{A}_T Interval availability is the time average of A(t) during intervals of length T, and is readily obtained from the equation for the average value of a function.



$$\overline{A}_{T} = \frac{\int_{0}^{T} A(t)dt}{\int_{0}^{T} dt} = \frac{1}{T} \int_{0}^{T} A(t) dt$$

Alternately, it may be considered that there is some probability distribution h(t) on demand time.

Then the interval availability is given by

$$E[A(t)] = \int_{0}^{\infty} A(t) h(t) dt$$

In the special case where h(t) is uniform on the interval [0,T]

$$h(t) = \frac{1}{T} \qquad 0 \le t \le T$$

$$0 \qquad t > T$$

then $\overline{A}_{T} = \frac{1}{T} \int_{0}^{T} A(t) dt$.

(3) A - the steady-state availability is the limiting interval availability as T -> •••

$$A = \lim_{t \to \infty} \frac{1}{T} \int_{0}^{T} A(t) dt$$

$$T \to \infty$$

It can also be shown that

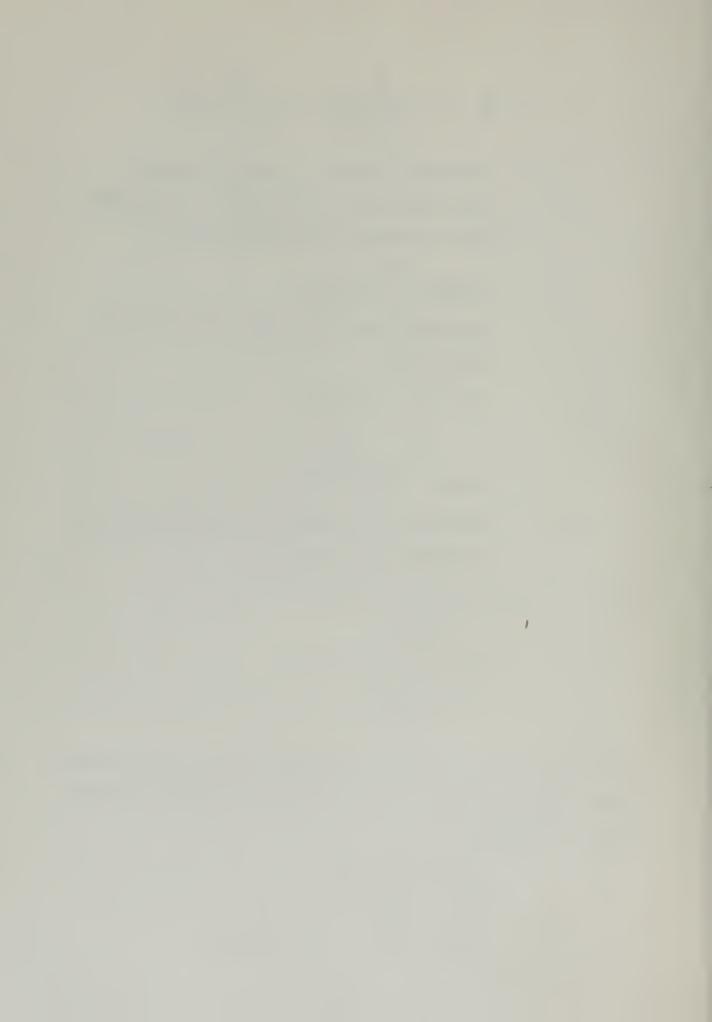
$$A = \lim_{t \to \infty} A(t)$$

If the components do not undergo checkout or repair its interval availability approaches zero. For systems with exponentially distributed failure times where

$$A(t) = e^{-\int t} \frac{1}{\sqrt{T}}$$

$$\overline{A}_{T} = \frac{1 - e}{\sqrt{T}}$$

$$\overline{A}_{T} \Rightarrow A \Rightarrow 0 \quad \text{as } T \Rightarrow \infty$$



Thus the steady state or long term uptime ratio of a component which does not undergo repair approaches zero.

If the component is subject to repair, its steady state availability is not zero. Assume that both failure and repair times are exponentially distributed with means $\frac{1}{\lambda}$ and $\frac{1}{\mu}$ respectively. Given that A(0) = 1 i.e., component available at start of mission, then

$$A(t) = \frac{\mu}{\mu + \lambda} + \frac{\lambda}{\mu + \lambda} e^{-t(\mu + \lambda)}$$

$$\mu = \text{repair rate}$$

$$\lambda = \text{failure rate}$$

$$hen \quad A = \lim_{t \to \infty} A(t) = \frac{\mu}{\mu + \lambda}$$

$$= \frac{\text{MTTF}}{\text{MTTF} + \text{MTTR}}$$

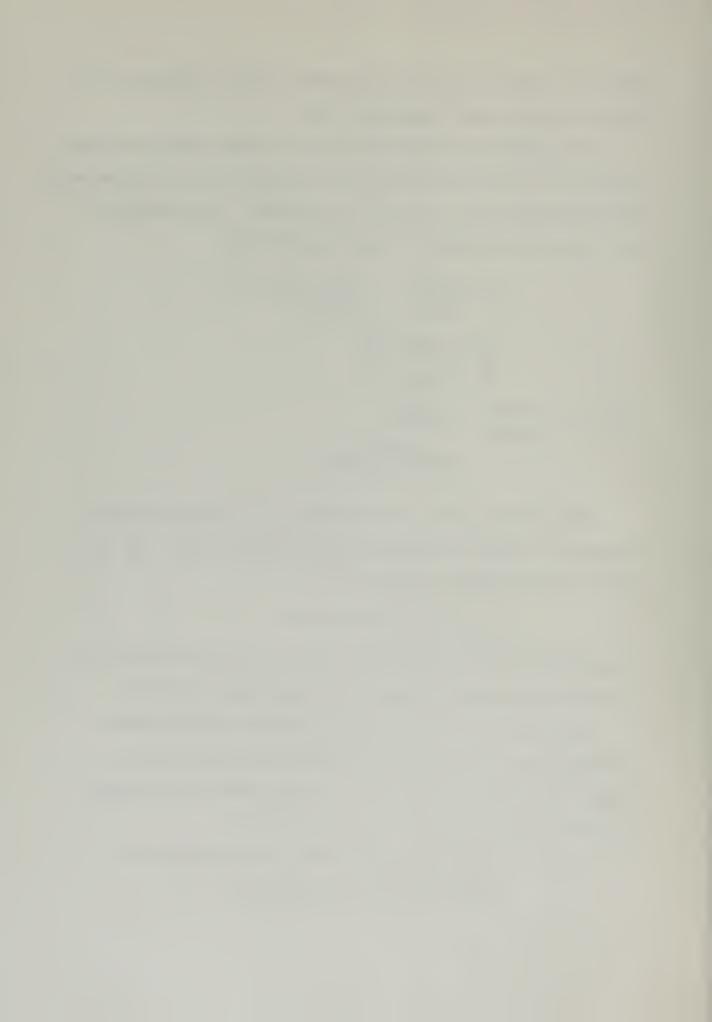
Catron [2] has shown that exact values of A(t) can be and were computed for repair distributions other than exponential. The use of interval availability as $t \rightarrow \infty$ i.e.,

$$\overline{A}_{T} = \frac{1}{\overline{T}} \int_{0}^{T} A(t) dt \rightarrow A$$

to approximate the true availability over the first mission time as is currently being done in practice, is a conservative procedure.

The amount by which the limiting interval availability underestimates the true availability during the first mission time is dependent upon the limiting value, i.e., the lower limiting value, the more it underestimates the true availability.

(4) A_S - System availability. For a series system the system availability is defined as:



$$A_{S} = \frac{\mathcal{K}}{II} \quad A_{i}$$

where A_i is the availability of the component of type i. $i = 1, 2, \cdots K$

B. PROBLEM

The problem considered in this paper is to check the accuracy of a proposed lower confidence limit procedure $\hat{A}_{SL(\propto)}$ for \hat{A}_{S} based upon component failure time data and repair time data.

(1) When assuming exponential failure rate \propto i and exponential repair rate β then A is given by

$$A_{i} = \frac{\beta i}{\beta i + \alpha i}$$

The following ranges of the failure rate (\propto i) and repair rate (β i) has been used in the simulation.

$$0.005 \leq \alpha_i \leq 0.01$$

$$3.0 \leq \beta_{i} \leq 10.0$$

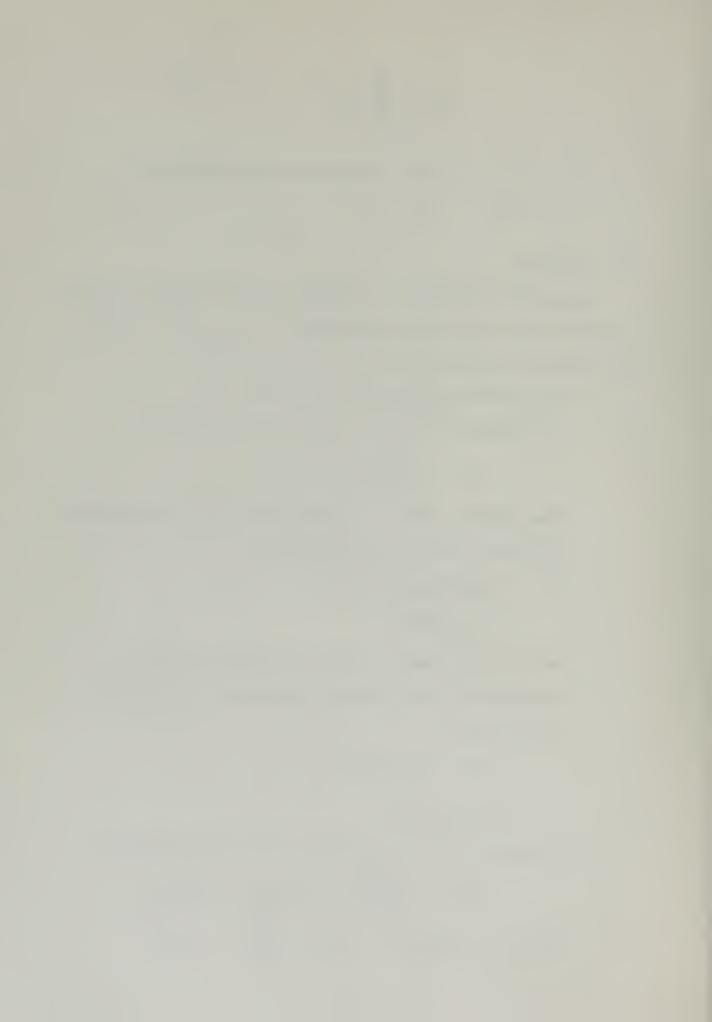
which implies that the ranges of MTTF of component i, denoted by λ_i and MTTR of component i, denoted by μ_i are as follows.

$$100 \leq \lambda_i \leq 200$$

$$\frac{1}{10} \leq \mu_i \leq \frac{1}{3}$$

(2) The equation of component availability can be written as

$$A_{i} = \frac{\beta i}{\beta i + \alpha i} = \frac{1}{1 + \frac{\alpha i}{\beta i}} = \frac{1}{1 + \alpha i \mu i}$$
but
$$A_{i} = \frac{1}{1 + \alpha i \mu i} = 1 - \alpha i \mu i + (\alpha i \mu i)^{2} - (\alpha i \mu i)^{3} + \cdots$$



provided the series converges, which it does since $\alpha_i \mu_i < 1$. Since the product $\alpha_i \mu_i$ is very small, in this case $0.0005 \le \alpha_i \mu_i \le 0.00333...$

$$A_{i} = \frac{1}{1 + \alpha_{i} \mu_{i}} = 1 - \alpha_{i} \mu_{i}$$
Then $A_{S} = \prod_{i=1}^{K} A_{i} = \prod_{i=1}^{K} (1 - \alpha_{i} \mu_{i}) = 1 - \sum_{i=1}^{K} \alpha_{i} \mu_{i}$

Since \propto_i and μ_i will have small variance we shall apply the central limit theorem, and fit a normal distribution to

$$A_{S} = 1 - \sum_{i=1}^{K} \angle i \mu i$$
Then $A_{S} = 1 - \sum_{i=1}^{K} \hat{A}_{i} \hat{\mu}_{i}$
Assuming that $A_{S} \sim N \left[A_{S}, \frac{1}{i=1} \left(\frac{n_{i}+1}{n_{i}} \frac{\angle i \mu_{i}}{\frac{N_{i}}{N_{i}}} \right) \right]$
implies that $A_{S} = 1 - \sum_{i=1}^{K} A_{S} = 1 - \sum_{i=1}^{$

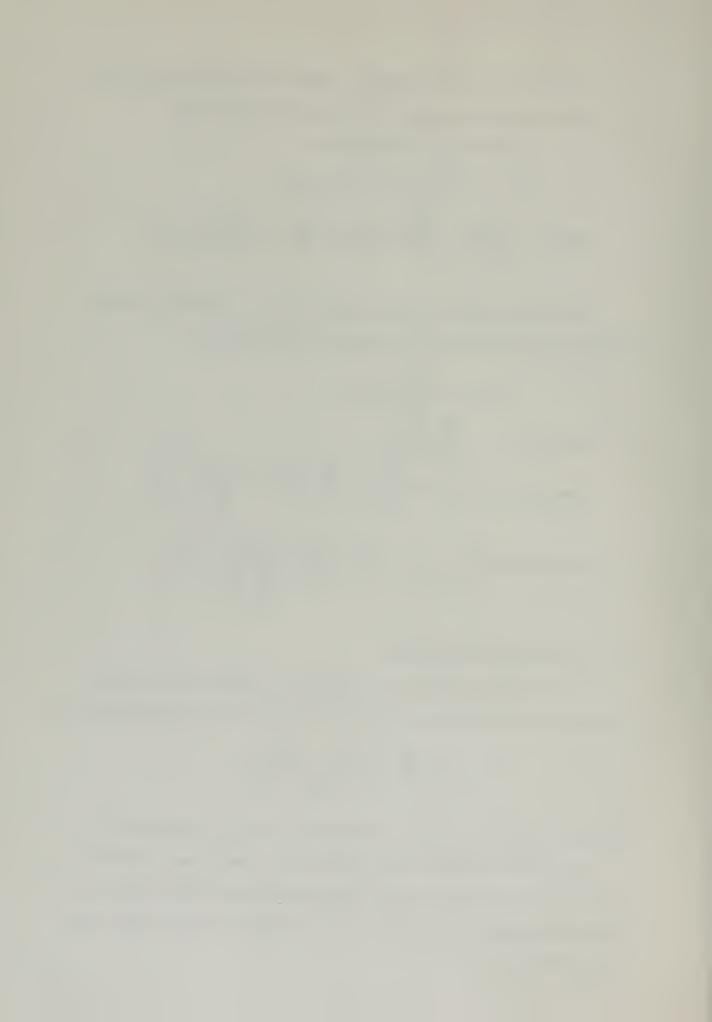
C. ALTERNATIVE METHOD

An alternative method was investigated, which fitted a 2 parameter gamma distribution by method of moments to the distribution of

$$A_{S} = \frac{K}{II} \hat{A}_{i} = \frac{K}{II} \frac{\hat{\beta}_{i}}{\hat{\beta}_{i} + \hat{\alpha}_{i}}$$

where \propto i and β i are the estimates of \propto i and β i respectively.

Under the assumption that time to failure and time to repair of the components were distributed exponentially with failure rate α_i and repair rate β_i respectively. The following approach was used to make the fit:



$$-\ln A_{S} = \sum_{i=1}^{K} -\ln \left(\frac{\beta i}{\beta_{i} + \alpha_{i}}\right) = \sum_{i=1}^{K} -\ln \left(1 - \frac{\alpha i}{\alpha i + \beta i}\right)$$

$$Q_{i} = 1 - A_{i} = 1 - \frac{\alpha i}{\alpha i + \beta i}$$

Thus by Taylor's expansion

$$-\ln A_{s} = \sum_{i=1}^{K} -\ln(1-\Omega_{i}) \stackrel{!}{=} \sum_{i=1}^{K} (\Omega_{i} + \frac{\Omega_{i}^{2}}{2})$$

$$\sum_{i=1}^{K} (\Omega_{i} + \frac{\Omega_{i}^{2}}{2}) = \sum_{i=1}^{K} \forall_{i} = \forall_{s}$$

$$\sum_{i=1}^{K} (\Omega_{i} + \frac{\Omega_{i}^{2}}{2}) = \sum_{i=1}^{K} (\Omega_{i} + \Omega_{i}^{2})$$

$$\sum_{i=1}^{K} (\Omega_{i} + \frac{\Omega_{i}^{2}}{2}) = \sum_{i=1}^{K} (\Omega_{i} + \Omega_{i}^{2})$$

$$\sum_{i=1}^{K} (\Omega_{i} + \Omega_{i}^{2}) = \sum_{i=1}^{K} (\Omega_{i} + \Omega_{i}^{2})$$
a_i and b_i are choosen so that

$$E(\sum_{i}) = \sum_{i}$$

and

$$\hat{Q}_{i} = 1 - \hat{A}_{i} = 1 - \frac{\hat{A}_{i}}{\hat{A}_{i} + \hat{A}_{i}}$$

where X_i and A_i are the estimate of X_i and A_i respectively.

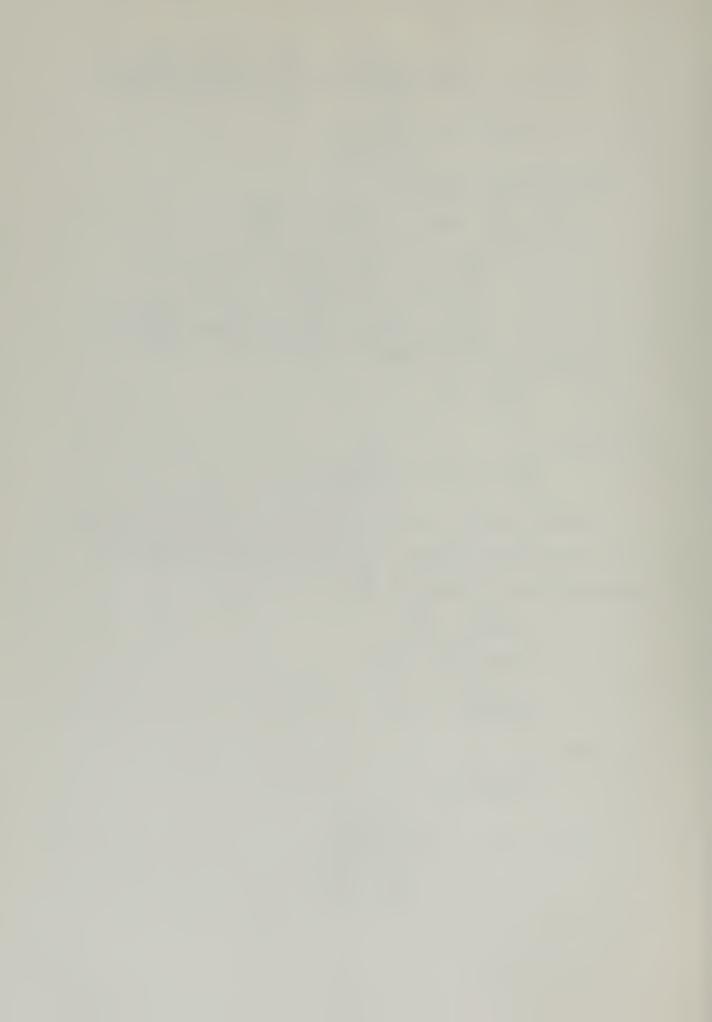
The 2 parameter gamma, Γ (Γ , \checkmark) was fitted by method of moments to the distribution of $\widehat{\gamma}_{5}$.

$$E(\cancel{y}_5) = \frac{r}{r}$$

$$Var(\cancel{y}_5) = \frac{r}{r^2}$$

$$\frac{E(\cancel{y}_5)}{Var(\cancel{y}_5)} = \frac{r}{r}$$

Therefore
$$\frac{E^{2}(\hat{Y}_{5})}{Var(\hat{Y}_{5})} = \Gamma$$
Then
$$\hat{\Gamma} = \frac{E^{2}(\hat{Y}_{5})}{Var(\hat{Y}_{5})}$$



by replacing all
$$\alpha_i$$
 and β_i by $\hat{\alpha}_i$ and $\hat{\beta}_i$.

Then $2 \vee \hat{\gamma}_s$ is distributed $\hat{\gamma}_s^2$.

$$- \omega_{A_S} = \hat{\gamma}_s = \hat{\gamma}_s$$

$$A_S = e^{-\frac{\pi}{2}}$$

From this assumption a one sided lower 100(1 - \propto)% C. L. for $^{\rm A}{\rm S}$ was computed.

i.e.,
$$P(2V\hat{Y}_{s} \geq \chi_{x,2r}^{2}) = 1 - \infty$$

$$P(e^{-\frac{r}{Y}} \geq exp(\frac{-\hat{Y}_{s}[2\hat{r}]}{\chi_{x,[2\hat{r}]}^{2}})) = 1 - \infty$$

$$P(A_{s} \geq \hat{A}_{sL}(\infty) = 1 - \infty$$

where $[2 \stackrel{\frown}{\Gamma}]$ = smallest integer greater than or equal to $2 \stackrel{\frown}{\Gamma}$. The procedure showed to be inaccurate due to small values of Γ .

D. PROCEDURE

The procedure used to fit a normal distribution to $1 - \sum_{i=1}^{n} \sqrt{i}$ as described in B is based on computer simulation.

The computations of the estimates of $\propto_{i's}$ and $\beta_{i's}$ (1=1,2,...k) are based on the assumption that we are given a series system with K types of components.

- V_{ij} time to failure of the j-th component of type i. The time to failure is assumed exponential with failure rate \propto .
- T_{ij} time to repair of the j-th component of type i. The time to repair is assumed exponential with repair rate β_i .

Vij and Tij are independent.



(1) A random sample of size N_i components of type i are tested until failure or a specified planned test time (PTT).

 $\boldsymbol{T}_{\mbox{oij}}$ is the PTT of the j-th $% \boldsymbol{J}_{\mbox{omponent}}$ component of type i

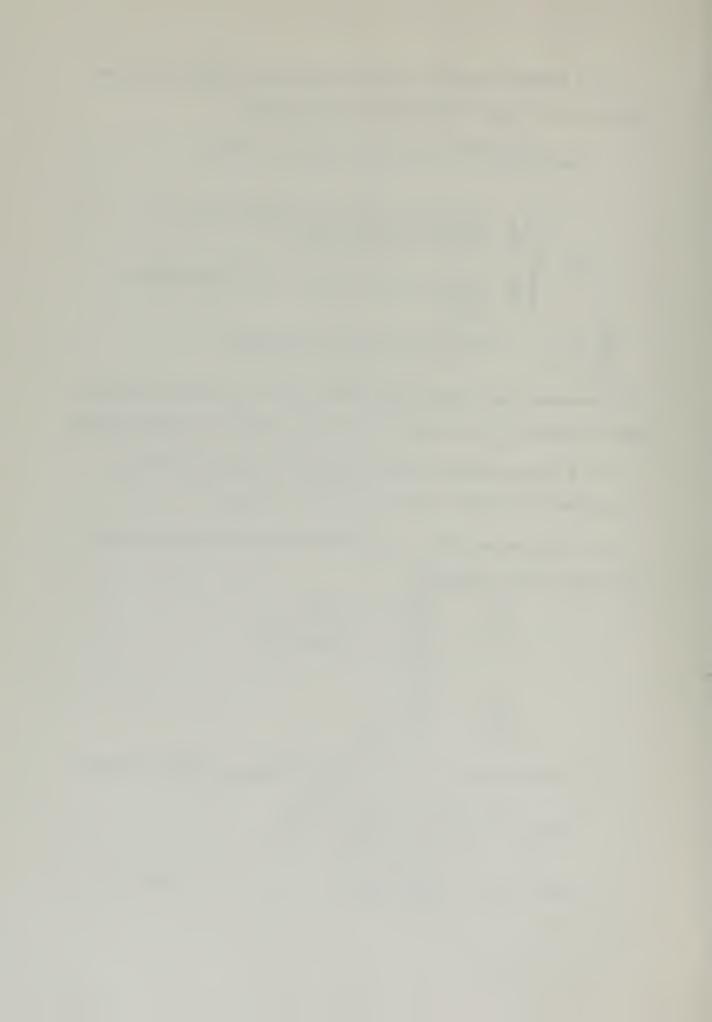
The components were tested against the (PTT) Toij and the observed operating times Voij set equal to Vij or Toij whichever is the smaller.

- (2) A random sample of size n_i of failed components of type i are repaired with repair times T_{i1} , T_{i2} , ..., T_{in_i} .
- (3) Using the test data described in C 1 and 2 the estimates of $\underset{i}{\swarrow}$ and $\underset{i}{\swarrow}$ were computed.

$$\hat{\chi}_{i} = \frac{\sum_{j=1}^{N_{i}} \chi_{i,j}}{\sum_{j=1}^{N_{i}} V_{o,i,j}} \left(\frac{2 N_{i}}{2 N_{i} + 1} \right)$$

$$\hat{L}_{i} = \sum_{j=1}^{n_{i}} T_{ij}$$

(4) With the data in C-3 1000 values of $A_{SL(\ll)}$ were computed.



Z is the $100(1-\alpha)$ th percentile point of the N(0,1) distribution.

E. ANALYSIS

The generated values of $^{\rm A}_{\rm S\,L(}$ \propto) was ordered from lowest to highest and the 1000(1- \propto)% one was compared to $^{\rm A}_{\rm S}$.

The ${\rm \stackrel{\wedge}{A}_{SL(\propto)}}$ was also compared to ${\rm \stackrel{\wedge}{A}_{S}}$ and the probability of success was computed by the formula

$$P(A_{SL(X)} \leq A_{S})$$

based on the 1000 values of $^{\wedge}_{SL(\, \, \swarrow \, \,)}$.

The results are shown in Appendix A for the different combinations of \propto , β , N_i , n_i and T_{oij} . To avoid fluctuations due to the random number generator, the values of $A_{SL(\propto)}$ at different \propto 's was computed by only changing Z_{\propto} .



III. ACCURACY RESULTS

By changing the values of the input parameters the accuracy of the procedure was checked. The following conclusions were made based on the simulation:

- a. Use of the steady state availability of the system (A_S) to approximate the estimated availability $A_{S L(\not X)}$ over the mission time is a conservative procedure.
- b. The amount by which the steady state availability differ from the estimated $100(1-\propto)$ the percentile point is very small.

Example

Case 3 - Appendix A.

With the given parameters the systems steady state availability is computed, giving

$$A_{S} = \frac{K}{11} \frac{\beta i}{\beta i + \alpha i} = .9925$$

The computed 95th percentile point of the distribution of $^{\wedge}_{S L(0.05)}$ is equal to .9931

Thus,
$$|A_S - \hat{A}_{S(950)}| = |.9925 - .9931| = \frac{.0006}{.9925}$$

Similarly the 90th and 80th percentile points of the distribution of $^{\Lambda}_{A_{SL(.1)}}$ and $^{\Lambda}_{SL(.2)}$ are .9930 and .9929 respectively. By increasing the number of items which are life tested from 50 to 100 and keeping the other parameters constant, we get the result given by case 5. That is,

$$A_S = .9925 \text{ (as before)}$$

$$A_{S L(0.05)} = .9928$$



This implies that

$$\left| A_{S} - A_{S(950)} \right| = \left| .9925 - .9928 \right| = \frac{.0003}{.0003}$$

In this case $A_{S(900)}$ and $A_{S(800)}$ = .9926 which implies that

$$\left| A_{S} - \stackrel{\wedge}{A_{SL(\infty)}} \right| = .0001$$

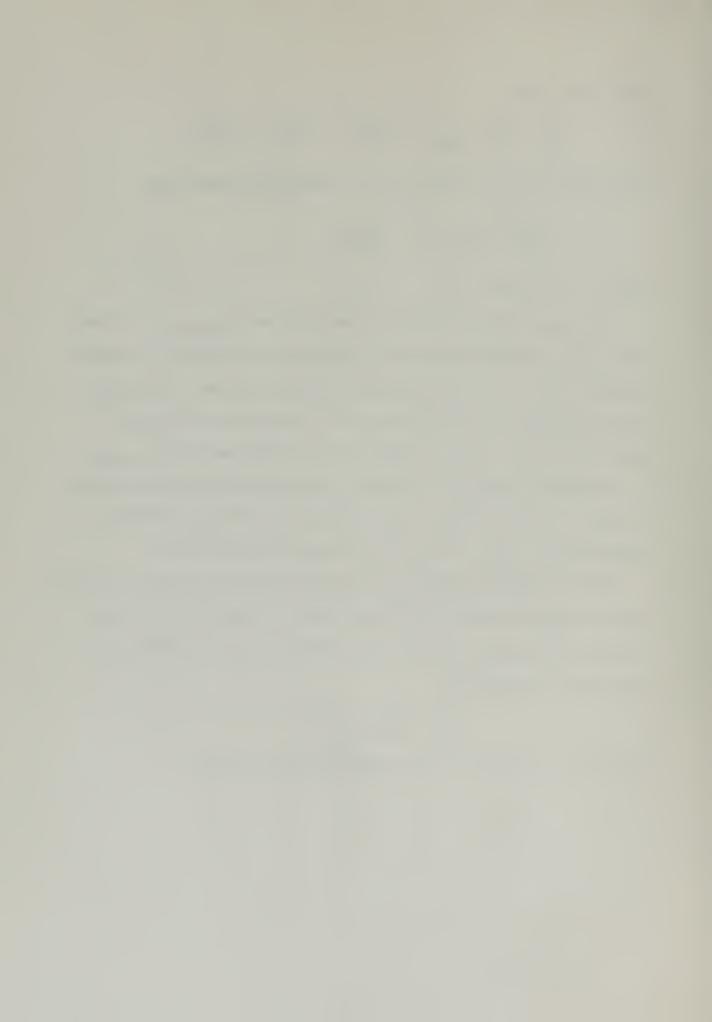
for $\cancel{\times}$ = 0.1 and $\cancel{\times}$ = 0.2

The true levels of confidence associated with ${\rm A_{SL}}(\propto)$ as a lower $100(1-\propto)$ confidence limit for ${\rm A_S}$ are given in columns 10 through 12 for $\propto = .05$, .10, .20 respectively. That is for Case 3, ${\rm A_{SL}}(.05)$ is an 90.8% lower confidence limit for ${\rm A_S}$ rather than 95% lower confidence limit. This is a measure of the inaccuracy of ${\rm A_{SL}}(.05)$ as a lower 95% confidence limit for ${\rm A_S}$ for the given parameter values in Case 3. Likewise ${\rm A_{SL}}(.10)$ is really a 84.5% lower confidence limit and ${\rm A_{SL}}(.20)$ is really a 76.9% lower confidence limit.

This variation between the true level of confidence and the proposed level of confidence (e.g., 90.8% vs. 95%) are due to the very small variance of $^{A}_{SL(\mbox{\ensuremath{\square}}\mbox{\ensuremath$

$$\left|A_{S} - A_{S(1-\infty)}\right|$$

are better measures for the accuracy of this procedure.



APPENDIX A

Result of the Simulation

_																			
12	$P(\widehat{A}_{S(.2)} = A_S)$.7360	. 7060	.7690	.7320	. 7930	.7540	.7640	.7600	.7660	.7970	.7750	.7400	.8080	.7680	.7980	.7730	.8040	.8230
11	$P(\widehat{A}_{S(.1)} \leq A_S)$	0778*	.7970	.8450	.8120	.8830	.8570	.8540	.8470	.8670	.8840	.8560	.8150	0968.	.8610	.8740	.8620	.8930	.9920
10	$P(\widehat{A}_{S(0.05)} = A_S) P(\widehat{A}_{S(.1)} = A_S) $	0906°	.8560	.9080	.8660	.9300	.9070	.9150	.9040	.9270	.9270	.9110	.8720	.9340	0806.	.9330	0606*	.9510	.9370
6	^A s(800)	.9930	7866.	.9929	. 9934	.9926	.9928	.9854	.9858	.9854	.9851	.9762	6226.	.9752	.9761	.9514	.9526	.9512	.9505
8	As(900)	.9931	.9942	.9930	.9937	.9926	.9930	.9857	.9861	.9854	.9853	.9976.	0626.	.9754	7976.	.9523	.9538	.9514	.9511
7	As(950)	.9934	.9953	.9931	7766.	.9928	.9934	.9858	7986.	.9854	.9859	.9772	.9814	.9761	.9780	.9526	.9548	.9512	.9529
9	AS	.9925	.9925	.9925	.9925	.9925	.9925	.9851	.9851	.9851	.9851	.9753	.9753	.9753	.9753	.9513	.9513	.9513	.9513
5	Toij	5.0	2.0	5.0	2.0	5.0	2.0	5.0	2.0	5.0	2.0	5.0	2.0	5.0	2.0	5.0	2.0	5.0	2.0
7	n,	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15
3	Z,	30	30	20	20	100	100	20	20	100	100	20	20	100	100	20	20	100	100
2	Bi	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0
1	j.	.005	.005	.005	.005	.005	.005	.010	.010	.010	.010	.005	.005	.005	.005	.010	.010	.010	.010
Column	Case	П	2	က	7	2	9	7	8	6	10	11	12	13	14	15	16	17	18



```
09/41/50
                                                                                                                                                DATF = 71046
                                                                                     MAIN
1 EVEL
                   1.8
                DIMENSION_ALEA(15), BETA(15), V(15,100), X(15,100), T(15,60), 14HAT(15), YHAT(15), N(15), NI(15), ASI(1000), ASZ(1000), ASZ(1000), ASZ(1000), ASZ(1000)
                    MA=1(00
                    K=15
                    Z05=1.645
Z10=1.282
                   Z10=1.202

720=0.842

IX=9345/3

FCRMAT(2F10.5,2I10)

DO 1000 I=1,X

READ (5,100) ALFA(I),BETA(I),N(I),NI(I)

CENTINUE
       100
     1000
                    X = () \cdot ()
                   X2=0.0

X3=0.0

KRITE(5,500)

FCRMAT(///,33X,'ASHATO5',3X,'ASHATIC',3X,'ASHAT20',///)

FCRMAT(30X,3F10.4)
                   DC 7(00 I=1,K

AS=AS*BFTA(I)/(ALFA(I)+BETA(I))

DC 8C00 M=1,MN

DC 2000 I=1,K

1A=N(I)

DC 3C00 J=1,FN

X(I,J)=C.0

(ALL RANDU(IX,IY,YFL)

IX=IY

V(I,J)=-1,0
        6CC
     7000
                   | X=IY

V(I,J)=-1.0/ALFA(1)*ALFG(YFL)

IF(V(I,J).LT.PTT(I,J)) X(I,J)=1.0

IF(V(I,J).GT.PTI(I,J)) V(I,J)=PTT(1,J)

CCNTIMUE

CENTIAUE

CENTIAUE

DC 3333 I=1,K

NAI=NI(I)

EC 4444 J=1.NMI
      3000
      2000
                      CALL RANDUCIX, IY, YEL)
                       1 > = [ Y
                     TY=[Y
T(I,J)=-1.0/6ETA(I)*ALOG(YFL)
CCNTIAUE
CENTIAUE
VA=0.0
AYHAT=0.0
DG 4000 I=1,K
SUYI=0.0
SUMXX=0.0
                     SUMXX=0.0

SUMV=C.0

AA=N(I)

DO 5(G) J=1,AN

SUMX=SUMXX+X(I,J)

SUMV=SUMV+V(I,J)

SUMT=SUMT+PIT(I,J)

CCNTINUF

SUMTT=0.0

AAI=NI(I)

DO 5555 J=1,AAI

CCNTINUF

XA=N(I)
       5000
       5555
                        XN = N(I)

XN I = NI(I)

A HAT (I) = SUMXX/SUMV*(2.0*XN)/(2.0*XN+1.0)

YHAT (I) = SUMIT/XNI
                       YHAT(I) = SUMTT/XNI
VA=VA+ MHAT(I) *YHAT(I) **2/SUMT
AYHAI=AYHAI+AHAI(I) *YHAI(I)
CONTINUE
ASHAI=1.0-AYHAI
ASI(M) = ASHAI+SORT(VA) * ZOS
AS2(M) = ASHAIT-SORT(VA) * ZIO
AS3(M) = ASHAIT-SORT(VA) * ZIO
IF(AS.GF.ASI(M)) XI=XI+1.0
IF(AS.GF.AS2(M)) X2=X2+1.0
IE(AS.GE.AS3(M)) X3=X3±1.0
       4000
```



09/41/50

```
__BJOO _CCATINUE __
    DC 8800 L=1,999
    JJ=L+1

DO 8°CO J=JJ,1000

IF(AS1(L).LT.AS1(J)) GO TO 800

TEMP1=AS1(L)
XMK=MN
PROB1=X1/XMU
PROB2= (2/X M)
Pack3= (3/X M)
 2, F7.4)
STCP
    END
```

MAIN



APPENDIX C

Derivation of Mean and Variance of A_S

$$A_{S} = \prod_{i=1}^{K} A_{i} = \frac{ik}{II} \frac{\beta_{i}}{\beta_{i} + \alpha_{i}} = \frac{k}{II} \frac{1}{1 + \alpha_{i}}$$
but
$$\frac{1}{\beta_{i}} = \mu_{i} = MTTR$$

The MTTR should always be less than one mission unit, which implies that

$$\alpha_i \mu_i < \alpha_i$$

thus

$$A_i = \frac{1}{1 + \alpha_i \mu_i} = 1 - \alpha_i \mu_i + (\alpha_i \mu_i)^2 - (\alpha_i \mu_i)^3 + \cdots$$

provided the series converges which it does since $\alpha = 1$

Since the product Xi Luis very small

$$A_{i} = \frac{1}{1 + \alpha_{i} \mu_{i}} = 1 - \alpha_{i} \mu_{i}$$

Thus

$$A_{S} = \prod_{i=1}^{K} (1 - \lambda_{i} \mu_{i}) = 1 - \sum_{i=1}^{K} \lambda_{i} \mu_{i}$$

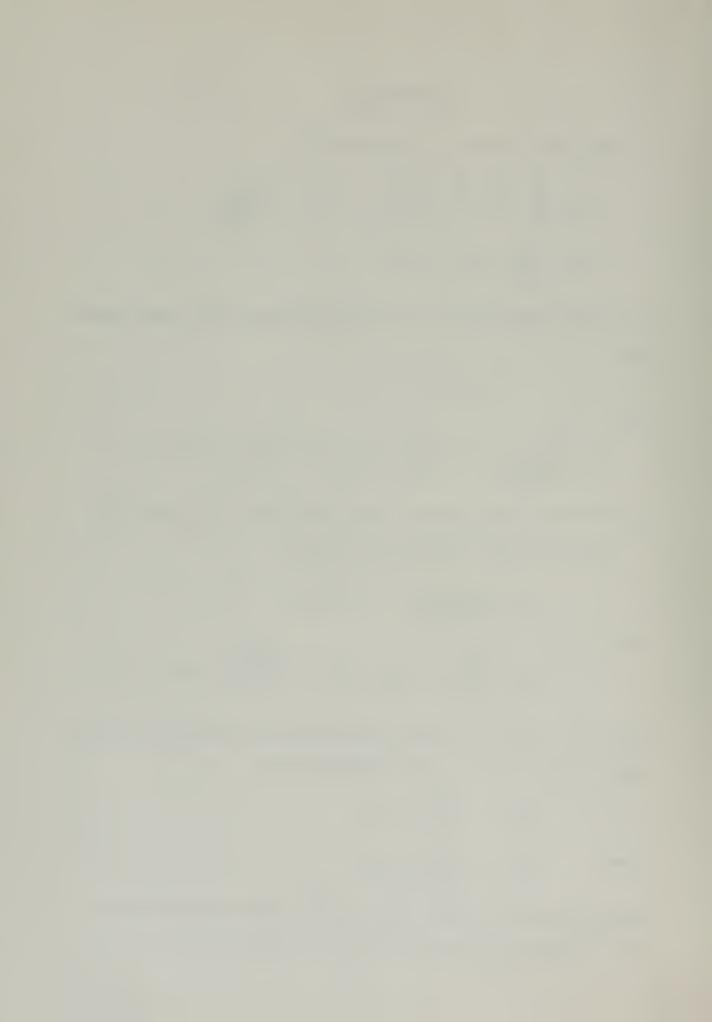
Since $\stackrel{\frown}{\alpha}_{i}$ and $\stackrel{\frown}{\beta}_{i}$ will have small variance we shall apply the central limit theorem, and fit a normal distribution to

$$A_{S} = 1 - \sum_{i=1}^{K} \langle x_{i} \rangle_{Li}$$

Thus

$$A_{S} = 1 - \sum_{i=1}^{K} \hat{\lambda}_{i} \hat{\mu}_{i}$$

when it is the estimated MTTR and \propto is the estimated failure rate. Based on the assumption that time to failure and time to



repair is distributed exponentially we can find the mean and variance of As.

$$E(\hat{A}_{S}) = 1 - \sum_{i=1}^{K} E(\hat{\mathcal{X}}_{i}) E(\hat{\mu}_{i}) = 1 - \sum_{i=1}^{K} \mathcal{X}_{i} \mu_{i} = A_{S}$$

By using the equations given by OD 29304 [3], for $\stackrel{\checkmark}{\alpha}$; and $\stackrel{\checkmark}{\alpha}$; we can find the variance for A

$$\stackrel{\wedge}{\swarrow}_{i} = \frac{\text{\# of failures}}{\text{\# of test time}} \cdot \frac{2 N_{i}}{2 N_{i} + 1}$$

$$\stackrel{\wedge}{E(\stackrel{\wedge}{\swarrow}_{i}) = \stackrel{\wedge}{\swarrow}_{i}}$$

$$\stackrel{\wedge}{Var}(\stackrel{\wedge}{\bigtriangleup}_{i}) = \frac{\stackrel{\wedge}{\swarrow}_{i}}{2 t_{ij}}$$

where $\sum_{i=1}^{N_c} t_{i,j}$ = sum of all planned test times on component i (in mission units).

$$\hat{\mu}_{i} = \frac{\text{# of repair times}}{n_{i}} = \frac{\pi_{i}}{\sum_{j=1}^{n} T_{i,j}}$$

Thus
$$Var(\hat{\mu}_{i}) = \mu_{i}$$

$$Var(\hat{\mu}_{i}) = \frac{\lambda i}{m_{i}}$$

$$= Var(\hat{\lambda}_{S}) = Var(1 - \frac{\kappa}{2} \hat{\lambda}_{i} \hat{\mu}_{i})$$

$$= Var(\frac{\kappa}{2} \hat{\lambda}_{i} \hat{\mu}_{i})$$

$$= \frac{1}{2} Var(\hat{\lambda}_{i} \hat{\mu}_{i})$$

$$= \frac{1}{2} Var(\hat{\lambda}_{i} \hat{\mu}_{i})$$

$$= \frac{1}{2} (\hat{\lambda}_{i} \hat{\mu}_{i}) = \frac{1}{2} (\hat{\lambda}_{i} \hat{\mu}_{i}) - \hat{\lambda}_{i} \hat{\mu}_{i}$$

$$= \frac{1}{2} (\hat{\lambda}_{i} \hat{\mu}_{i}) = \frac{1}{2} (\hat{\lambda}_{i} \hat{\mu}_{i}) - \hat{\lambda}_{i} \hat{\mu}_{i}$$

$$= \frac{1}{2} (\hat{\lambda}_{i} \hat{\mu}_{i}) - \hat{\lambda}_{i} \hat{\mu}_{i}$$

$$= \frac{1}{2} (\hat{\lambda}_{i} \hat{\mu}_{i}) - \hat{\lambda}_{i} \hat{\mu}_{i}$$
But
$$E(\hat{\lambda}_{i}) = Var(\hat{\lambda}_{i}) + \hat{\lambda}_{i}^{2}$$

But

$$= \frac{\angle i}{\sum_{j} t_{ij}} + \alpha i$$



$$E(\hat{\mu}_{i}^{2}) = Var(\hat{\mu}_{i}) + \mu_{i}^{2}$$

$$= \frac{\mu_{i}^{2}}{\pi_{i}} + \mu_{i}^{2}$$

$$= \mu_{i}^{2} \left(\frac{\eta_{i}+1}{\eta_{i}}\right)$$

Therefore
$$Var(\hat{\lambda}_{i}, \hat{\mu}_{i}) = \left(\frac{\lambda_{i}}{Z_{tij}} + \lambda_{i}\right) \left(\frac{\lambda_{i}}{\mu_{i}}, \frac{\eta_{i+1}}{\eta_{i}}\right) - \lambda_{i}^{2}\mu_{i}^{2}$$

$$= \frac{\lambda_{i}\mu_{i}}{Z_{tij}} \cdot \frac{\eta_{i+1}}{\eta_{i}} + \lambda_{i}^{2}\mu_{i}^{2}, \frac{\eta_{i}+1}{\eta_{i}} - \lambda_{i}^{2}\mu_{i}^{2}$$

$$= \frac{\lambda_{i}\mu_{i}}{Z_{tij}} \cdot \frac{\eta_{i+1}}{\eta_{i}} + \lambda_{i}^{2}\mu_{i}^{2}, \frac{\eta_{i}+1}{\eta_{i}} - \lambda_{i}^{2}\mu_{i}^{2}$$

which implies that

$$Var(\hat{\lambda}_i \hat{\mu}_i) = \frac{\lambda_i \mu_i}{\sum_{j} t_{ij}} \cdot \frac{\Pi_i + 1}{\Pi_i}$$

Thus by (1)

$$var(\hat{A}_5) = \sum_{i=1}^{K} \frac{\angle i \mu_i^2}{\sum_{j=1}^{N_2} \lambda_{i,j}^2} \cdot \frac{\prod_{i=1}^{N_2} + 1}{\prod_{i=1}^{N_2} \lambda_{i,j}^2}$$

Since α_i and μ_i is approximately equal to α_i and μ_i respectively,

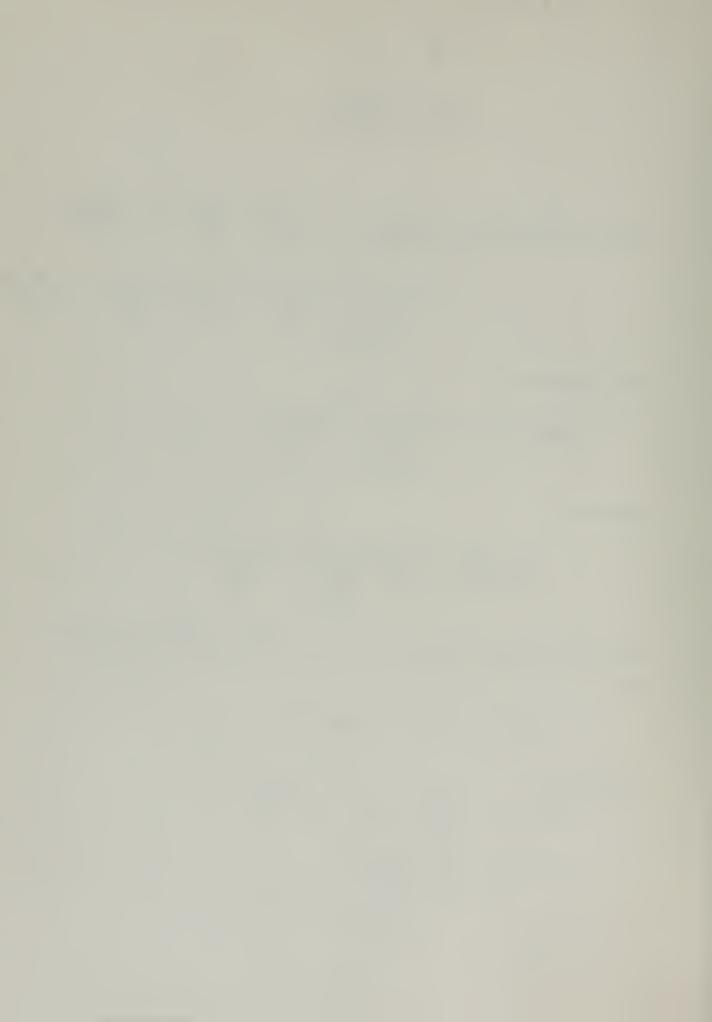
$$\frac{n_i + 1}{n_i} \rightarrow 1 \text{ as } n_i \rightarrow \infty$$

we can write

$$E(\hat{A}_{S}) = 1 - \sum_{i=1}^{K} \hat{\lambda}_{i} \hat{\mu}_{i} \stackrel{e}{=} \hat{A}_{S}$$

$$var(\hat{A}_{S}) = \sum_{i=1}^{K} \frac{\hat{\lambda}_{i} \hat{\mu}_{i}^{2}}{N_{i}^{2}}$$

$$\sum_{j=1}^{K} \frac{\hat{\lambda}_{j} \hat{\mu}_{i}^{2}}{N_{i}^{2}}$$



which implies that

$$\hat{A}_{S} \sim N(A_{S}, \text{ Var } (A_{S})) \stackrel{\epsilon}{=} N(A_{S}, \text{ Var } (\hat{A}_{S}))$$

$$= N\left(1 - \frac{1}{2}\hat{\alpha}_{i}\hat{\mu}_{i}, \frac{1}{2}\hat{\alpha}_{i}\hat{\mu}_{i}\right)$$

Thus by the above assumptions we can write

$$P\left(\frac{\hat{A}_{S} - A_{S}}{Var \hat{A}_{S}} \leq Z_{\infty}\right) = 1 - \infty$$

$$P\left(\hat{A}_{S} - A_{S} \leq Z_{\infty} \middle| Var(\hat{H}_{S})\right) = 1 - \infty$$

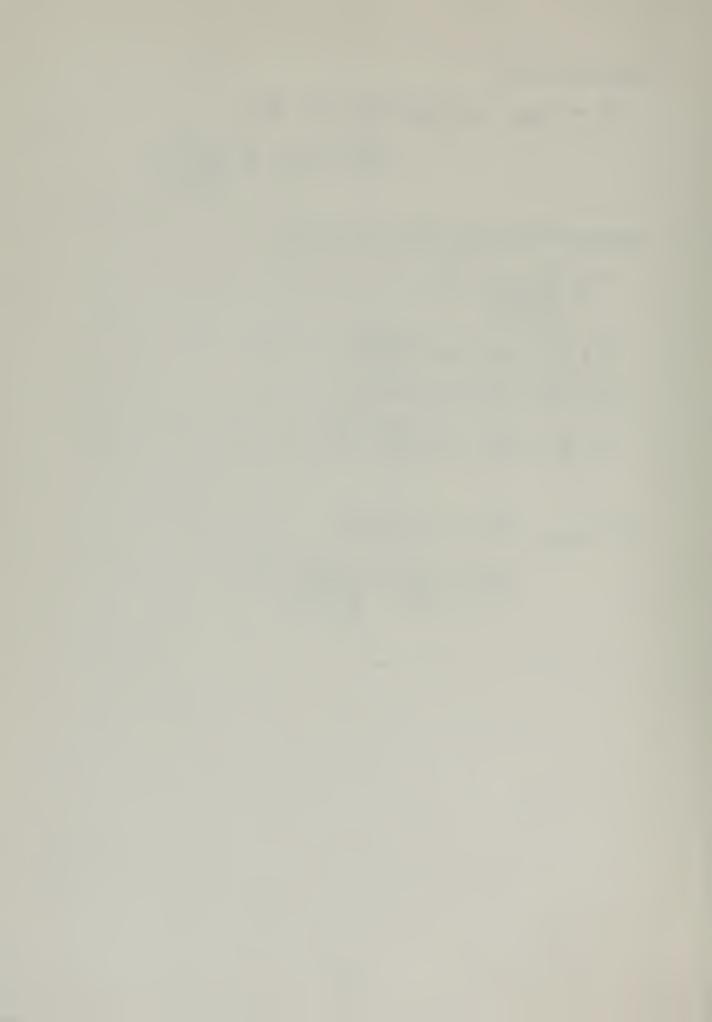
$$P\left(-A_{S} \leq -\hat{A}_{S} + Z_{\infty} \middle| Var(\hat{H}_{S})\right) = 1 - \infty$$

$$P\left(A_{S} \geq \hat{A}_{S} - Z_{\infty} \middle| Var(\hat{A}_{S})\right) = 1 - \infty$$

$$P\left(A_{S} \geq \hat{A}_{S} - Z_{\infty} \middle| Var(\hat{A}_{S})\right) = 1 - \infty$$

Thus
$$\hat{A}_{SL(x)} = \hat{A}_{S} - Z_{x} \sqrt{\text{Var}(\hat{A}_{S})}$$

$$= \hat{A}_{S} - Z_{x} \left(\sum_{i=1}^{K} \frac{\hat{A}_{i} \hat{A}_{i}}{\sum_{i=1}^{K} t_{c,i}} \right)^{\frac{1}{2}}$$



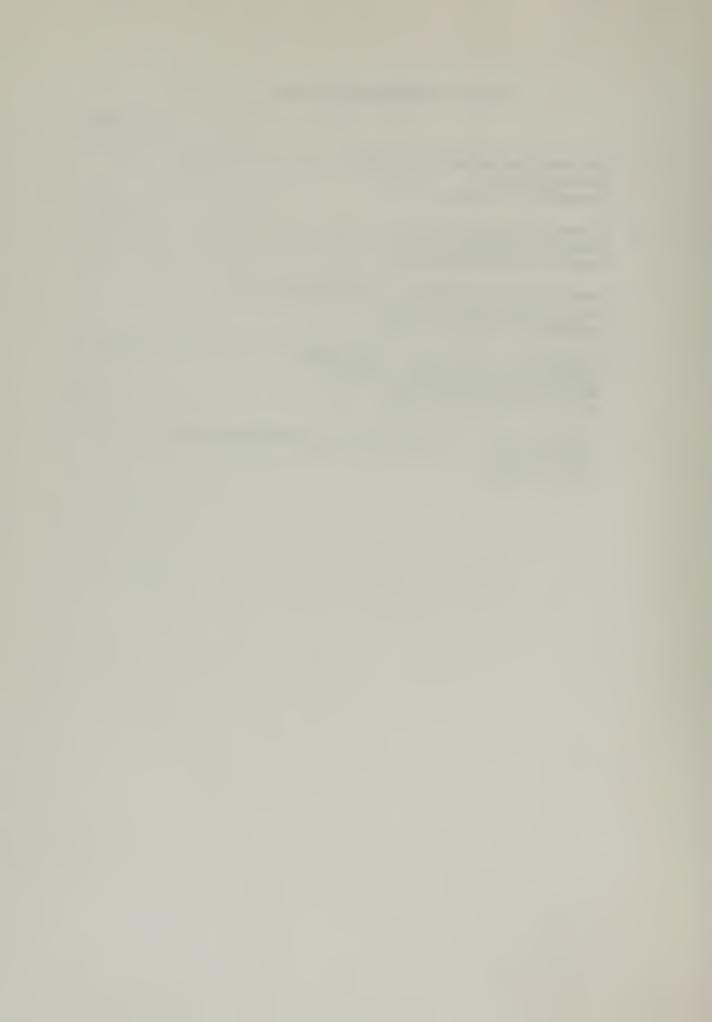
BIBLIOGRAPHY

- Department of the Navy, Strategic Systems Project Office, NAVORD OD 43251 <u>Availability Evaluation Program Manual</u>, 1 January 1970.
- 2. Catron, G. R., Investigation of the Accuracy of Using Steady
 State Results to Approximate Actual System Availability.
 M.S. thesis, U.S. Naval Postgraduate School, Monterey, April 1970.
- 3. Department of the Navy, Special Project Office, NAVWEPS OD 29304, Guide Manual for Reliability Measurement Program, 15 May 1965.



INITIAL DISTRIBUTION LIST

		No.	Copies
1.	Defense Documentation Center Cameron Station Alexandria, Virginia 22314		2
2.	Library, Code 0212 Naval Postgraduate School Monterey, California 93940		2
3.	Department of Operations Analysis (Code 55) Naval Postgraduate School Monterey, California 93940		1
4.	Professor W. M. Woods, Code 55Wo Department of Operations Analysis Naval Postgraduate School Monterey, California 93940		1
5.	LCDR Bjørnar J. Kibsgaard, Royal Norwegian Navy SST Oslo Mil Oslo 1, Norway	,	1



Security Classification					
DOCUMENT CONT	TROL DATA - R	& D			
(Security classification of title, body of abstract and indexing	annotation must be e	ntered when the	overall report is classified)		
1 ORIGINATING ACTIVITY (Corporate author)		28. REPORT SI	ECURITY CLASSIFICATION		
Naval Postgraduate School		Unclassified			
Monterey, California 93940		2b. GROUP			
3 REPORT TITLE					
An Accuracy Analysis of an Ad Hoc Low	er Confiden	re Limit l	Procedure for		
System Availability	er confident		10004410 101		
System Avallability					
4 DESCRIPTIVE NOTES (Type of report and, inclusive dates)					
Master's Thesis; September 1971					
5. AUTHOR(5) (First name, middle initial, last name)					
Bjørnar Johan Kibsgaard					
Djørnar sonan misogaara					
6 REPORT DATE	74. TOTAL NO. O	FPAGES	76. NO. OF REFS		
September 1971	26		3		
88. CONTRACT OR GRANT NO.	94. ORIGINATOR'S	REPORT NUM	BER(\$)		
·					
b. PROJECT NO.					
c.	9b. OTHER REPORT NO(5) (Any other numbers that may be assigned this report)				
d.	1		·		
10. DISTRIBUTION STATEMENT					
Approved for public release; distribution	on unlimited	•			
11. SUPPLEMENTARY NOTES	12. SPONSORING	ALLITARY ACT	VITY		
	Naval Postgraduate School Monterey, California 93940				

The accuracy of proposed lower confidence limits for system availability is analyzed. Random values of the lower 100(1- \propto)% confidence limit $^{A}SL(\propto$) for system availability are computed for a system whose failure density is exponential (\bigwedge) and whose repair density is exponential (\mathcal{L}_{ζ}). The system is modeled as an alternating renewal process. The 100(1-) th percentile point of the generated distribution of $A_{SL(X)}$ is compared with system

availability as a measure of accuracy for $^{A}{
m S}$ L(imes).

(PAGE 1) S/N 0101-807-6811

13. ABSTRACT

UNCLASSIFIED Security Classification



· Security Classification		UNCLASSIFIED Security Classification								
KEY WORDS	LIN	LINK B		LINK C						
KEY WONDS	ROLE	wT	ROLE	WT	ROLE	WT				
Accuracy										
System availability										
•		-								
			1							
				İ	1					
		1								

A-31409

26







Thesis K3965 127793

c.l Kibsgaard

An accuracy analysis of an ad hoc lower confidence limit procedure for system availability.

7 JAN 88

33403

Thesis K3965 117793

c.1 Kibsgaard An acc

An accuracy analysis of an ad hoc lower confidence limit procedure for system availability.

